

DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

# **CALIFORNIA INSTITUTE OF TECHNOLOGY**

PASADENA, CALIFORNIA 91125

## **ASPIRATIONS AND GROWTH: A MODEL WHERE THE INCOME OF OTHERS ACTS AS A REFERENCE POINT**

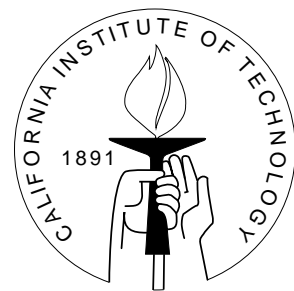
Francesco Bogliacino

Departamento de Economía and RISE Group - Universidad EAFIT

European Commission - JRC - IPTS

Pietro Ortoleva

California Institute of Technology



## **SOCIAL SCIENCE WORKING PAPER 1314R**

October 2009

Revised December 2010

# Aspirations and growth: a model where the income of others acts as a reference point

Francesco Bogliacino

Pietro Ortoleva

## Abstract

We study an OLG model in which the average income of the society acts as a reference point for the agents' utility on consumption. To model this we use the functional form developed in behavioral economics to study reference-dependence: prospect theory. We then assume that: 1) the utility function is convex in an interval before the reference point; 2) the utility function is not differentiable at the reference point, and it is steeper below than above the reference point. We argue that this reference-dependence causes the economy to admit multiple equilibria, and we show that in any of these equilibria in finite time the wealth distribution will become, and remain, either polarized or of perfect equality. We then study growth rates and show that, if we look at the equilibria with the highest growth, then the society that grows the most is the one that starts with perfect equality. If we look at the equilibria with the lowest growth for each economy, however, then the society with a *small* amount of initial inequality is the one that grows (strictly) the most, while a society with perfect equality is the one that grows the least. All of these growth rates are weakly higher than the growth rate of a corresponding economy without reference-dependence.

JEL classification numbers:

Key words: Aspirations, Interdependent Preferences, Reference-dependence, inequality and growth

# Aspirations and growth: a model where the income of others acts as a reference point

Francesco Bogliacino\*

Pietro Ortoleva

## 1. Introduction

### 1.1 *Basic Idea*

While the traditional approach in economic modeling is to assume that individual preferences are a fixed ordering irrespective of the environment in which agents operate, many arguments have been put forth in many branches of economics to question this assumption. On the one hand, for example, it has been frequently argued both in the development and in macro-economic literature that how much utility subjects assign to a certain bundle, or to a certain achievement, might fundamentally depend on the behavior of the rest of the society. In particular, individuals might form *aspirations* about what that they would like to accomplish based on the behavior they observe in others, like their parents, their neighbors, or the whole society. This feature could naturally have strong implications. For example, Appadurai (2005) and Ray (2006) suggest how aspirations could play both a positive and a negative role in the development of a country: they might induce subjects to ‘work hard’ to reach their aspirations; at the same time, individuals who are too far from reaching them might simply ‘give up.’

On the other hand, it is now an established finding in behavioral and experimental economics that the utility that subjects associate to the different available alternatives often depends on some reference point – some amount, or object, that subjects compare the available options with in order to make a choice.<sup>1</sup> These reference-effects have been shown to have an enormous impact on agent’s decisions, and have spurred the development of a large literature aimed at identifying the correct functional form to represent

---

\*Any opinion expressed in this paper is under full responsibility of the author and should never be interpreted as an official position by the European Commission.

<sup>1</sup>See among others, Tversky and Kahneman (1974), Kahneman and Tversky (1979), Tversky and Kahneman (1981), Samuelson and Zeckhauser (1988), Camerer (1995).

it.<sup>2</sup> Among these, one of the most well-known model is Prospect Theory, discussed in Kahneman and Tversky (1979) and Tversky and Kahneman (1991), whose functional form has three main features. First, it is reference-dependent – the utility of the agent depends on her reference point. Second, the utility function is “steeper for losses than for gains:” in particular, the utility function has a kink at the reference point, where the left derivative is higher than the right derivative – generating a behavior known as ‘loss aversion.’ Finally, the utility function is *concave* above the reference point, but *convex* below the reference point – a feature sometimes referred to as ‘diminishing sensitivity.’ While prospect theory is agnostic about the origin of the reference point, with few exceptions the literature has focused only on the cases in which the reference point is either agent’s past consumption, or her expectations.

The goal of this paper is to study the role of aspirations in development by merging these two different approaches. In particular, we follow the intuition suggested in the development literature about the role of the society in affecting subjects’ aspirations: we focus on the case in which the aspirations of each household are the average income of the rest of the society. At the same time, we use the functional form developed in behavioral economics, prospect theory, to model such reference-dependence.

## 1.2 Our results

We study a warm-glove overlapping generations model in which the utility that households derive in the second period of their lives depends on the average income of the other member of the society during that period,  $\bar{x}_t$ . Following prospect theory, we assume that the utility function is *convex* in an interval that ends with  $\bar{x}_t$ , and that it has a kink (it is not differentiable) at  $\bar{x}_t$ , where the left-derivative is higher than the right-derivative. Everything else in the model is entirely standard.

We obtain the following results. First, we argue that this economy has multiple equilibria, where the multiplicity is mostly due to the presence of these reference effects. Despite this multiplicity, however, all the equilibria have a common feature: there is a finite period  $T$  after which the wealth distribution of the economy is either of perfect equality or polarized, and so it will remain for all subsequent periods.<sup>3</sup> No other distribution is possible in the long run, and one of these distributions must be reached in finite time (as opposed to asymptotically, or in steady state). Furthermore, we show that whether the economy converges to one distribution or the other depends both on the initial distribution, and on the equilibrium that we are looking at. On the one hand,

---

<sup>2</sup>For theoretical behavioral models see Kahneman and Tversky (1979), Tversky and Kahneman (1991), Köszegi and Rabin (2006). For axiomatic models, see Chateauneuf and Wakker (1999), Masatlioglu and Ok (2005, 2008), Diecidue and Van de Ven (2008), Ok et al. (2009), Ortoleva (2010), and Wakker (2010).

<sup>3</sup>By a polarized distribution we understand a distribution with a ‘gap in the middle’, i.e. a wealth distribution such that there is an interval in the interior of the support which contains zero mass. The idea is that the wealthy will lay above this gap, while the poor will lay below.

every society with a non-degenerate initial distribution has an equilibrium in which the wealth distribution will become polarized in the long run. On the other hand, a society that has an initial distribution that is very ‘concentrated’ (the difference between the average and the poorest household is ‘small’) will *also* have an equilibrium in which the wealth distribution converges to perfect equality in finite time.

We then study how the initial distribution of endowments affects the relative growth of societies with aspirations. First, we show that if we focus on the equilibria with the highest growth for each economy, fixing the average income the economy that grows the most is the one with an initial distribution of perfect equality.

If we look at the equilibria with the lowest growth for each society, however, things might change considerably: the society that grows the most is the one with some initial inequality, but that admits an equilibrium of perfect equality. In particular, this economy will grow *strictly more* than an economy that starts from perfect equality – the latter will grow just like an economy without reference dependence in this equilibrium. That is, we show that if we focus on the equilibria with the lowest growth, then ‘a little bit’ of initial inequality is good for growth.

We also show that if the utility exhibits Constant Relative Risk Aversion, then any society with reference-dependence grows (weakly) more than a standard economy with the same initial endowment but without reference-dependence. Put differently, we show that the presence of reference-dependence in the form of aspirations cannot be bad for growth.

### 1.3 *Outline and Related Literature*

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model, the notion of equilibrium that we use, and describes the optimal choice of the households in this environment. Section 3 presents the consequences of this reference-dependence on the wealth distribution of the society in the long run, while Section 4 analyses those on growth. Section 5 concludes. The proofs appear in the appendix.

Our model is directly related to papers in two separate literatures. First, it is connected to the models that show the consequences of prospect-theory-like behavior in broad economic models; second, it is connected to the models in development economics that study the role of aspirations and their impact on growth. Within the first group, Koszegi and Rabin (2009) and Bowman et al. (1999) analyze the consequence of reference-dependent behavior in a consumption-saving model. As opposed to our work, however, their focus is on the case in which the reference point is the agent’s past behavior, instead of the average choice of the rest of the society. From this point of view, our model could be seen as providing another analysis of the consequences of reference-dependent behavior in broad economic setting, where a different origin for the reference point is considered.

Within the development literature, formal models that study the role of aspirations appear in Banerjee (1990), Genicot and Ray (2009), Dalton and Ghosal (2009 (Forthcoming)), and Mookherjee et al. (2010). Mookherjee et al. (2010) studies the case in which agents look at their ‘neighbors’ to form their aspirations – i.e. they focus on a ‘local’ origin of aspirations, instead of a society-wide origin like we do. (Furthermore, they use a functional form very different from prospect theory.) Genicot and Ray (2009) studies an OLG model in which subjects live one period and are reference-dependent, with a utility function, differentiable everywhere, which is convex in an interval right before the aspirations level. They first show that there exists an inverse U effect of aspirations on accumulation decision, proving the conjecture in Ray (2006). Then, focusing on the case in which aspirations are the average income of the society, they show that the support of the wealth distribution in steady state is generically finite, and that, under the condition that aspirations are, loosely speaking, ‘important enough,’ a distribution of perfect equality cannot exist. In addition, they also consider the case of ‘upward looking’ aspirations, i.e. the case in which each individual’s aspirations are the average income of those *above* her in the income distribution, and show that in this case continuous income distributions can exist in a steady state if and only if they are Pareto distributions. By contrast, in our paper we focus only on the case in which aspirations are the average income of the society, but consider a utility function which not only has an area of convexity, but also admits a kink at a reference point – directly following prospect theory and the notion of loss aversion. This feature allows us obtain our different results on the long run distribution of endowment: that the distribution becomes, and remains, either of perfect equality or polarized, therefore also admitting the case of continuous support. (In our model these features are reached in finite time, as opposed to in the steady state.) Furthermore, it also allows us to characterize the effects of the initial distribution on growth.

Finally, our work is naturally connected also with the literature in macroeconomics that is usually referred to as *keeping up with the Joneses*, that models agents whose utility for consumption depends also on the relative position of the agent in the wealth distribution (their status). (An analysis closer to the one in our work appears in Konrad (1992), Fershtman et al. (1996), Rauscher (1997), Corneo and Jeanne (2001), Cooper et al. (2001), Stark (2006), Hopkins and Kornienko (2006).) Their approach, however, is fundamentally different from ours for two reasons. First, these models assume that individuals care about their status using some additive or multiplicative functional form, which renders the model very different from assuming that subjects care about how their income relate to the average one using a prospect-theory form. Second, most of these works focus more on the properties of the accumulation path than on the implications on the wealth distribution in the long run – as evident from the fact that most use a representative agent. By contrast, the development of the wealth distribution, and its effects on growth, are the essential focus of our study.

## 2. A Model of Growth with Aspirations

### 2.1 Formal Setup

We study an economy with overlapping generations and warm glove preferences. The economy is populated by a continuum of size  $L$  of agents who live for two periods, make choices in the first period, generate an offspring exogenously in the second period, and have a bequest motive directly in the utility function.<sup>4</sup> Every agent is endowed with  $e_{it}$  of human capital and  $k_{it}$  of physical capital, which are determined by the decision taken by the previous generation. We assume constant return to scale in the accumulation of human capital, i.e. by investing  $e_{it+1}$  households obtain  $Ae_{it+1}$ , and without loss of generality we put  $A = 1$ . Since our interest is mostly related to developing countries, we impose that the access to credit market is restricted: individual consumers cannot borrow. (We will later argue that this assumption does not seem to be what is driving our results. See Remark 2.) By  $\beta \leq 1$  we denote the discount factor;  $\mu_t$  stands for the distribution of endowments in each period;  $r_t$  and  $w_t$  represent the interest rate and the wage. Finally, in the economy we have a representative firm with a production function  $F(K, H)$ .

The individual household maximizes:

$$\begin{aligned} \max_{c_{i,t}, x_{i,t+1}, e_{i,t+1}, k_{i,t+1}} & u(c_{it}) + \beta v(x_{i,t+1}, \bar{x}_{i,t+1}) \\ \text{such that} & \\ c_{i,t} + e_{i,t+1} + k_{i,t+1} & \leq w_t e_{i,t} + (1 + r_t) k_{i,t} \\ x_{i,t+1} & = w_t e_{i,t+1} + (1 + r_t) k_{i,t+1} \end{aligned} \tag{1}$$

As opposed to standard OLG problems, the agent's utility for the second period,  $v$ , can be different from the one of the first period,  $u$ . In particular, the former also depends on an additional term,  $\bar{x}_{i,t}$ , which we will interpret as the agent's *aspirations* for period  $t$ .

We start by imposing standard restrictions.

**Assumption 1.**  $u(\cdot)$  is increasing and concave and satisfies the Inada conditions.

**Assumption 2.** The production function  $F(K, H)$  is increasing, concave, homogeneous of degree one, and satisfies the Inada conditions.

Both requirements above are standard. In particular, agents have a standard utility function in the first period of their lives. (In Section 4 we will also consider the more specific case in which  $u$  exhibits Constant Relative Risk Aversion, CRRA.) The main feature of our model is the shape of  $v$ .

**Assumption 3.** There exist a  $H > 0$  such that for every  $\bar{x} \in \mathbb{R}_+$ , the following holds:

---

<sup>4</sup>The presence of an uncountable number of agents is inessential for our results. It is routine to show that everything we prove would hold true with finitely many agents.

1. *v is continuous and monotone*:  $v(\cdot, \bar{x})$  is strictly monotone and continuous;  $v(x, \cdot)$  is continuous for every  $x \in \mathbb{R}_+$ ;
2. *v moves like u when not close to the aspirations*: for all  $x \notin [\bar{c} - H, \bar{c}]$ ,  $v(x, \bar{x})$  is twice differentiable and  $\frac{dv(x|\bar{x})}{dx} = \frac{du(x)}{dx}$  ;
3. *v has a prospect-theory form*:
  - (a)  $v$  is strictly convex on  $(\bar{x} - H, \bar{x})$ ;
  - (b)  $\lim_{x \rightarrow \bar{x}^-} v(x, \bar{x}) > \lim_{x \rightarrow \bar{x}^+} v(x, \bar{x})$ .<sup>5</sup>

The idea behind Assumption 3 is that our agents have an *aspirations level*  $\bar{x}_i$ , which acts as a reference point in affecting their behavior. This reference point affects the second period utility  $v$  in two ways, both of which are the fundamental features of prospect theory (part (3) of Assumption 3):

1. *v is steeper for gains than for losses, and in particular it is not differentiable at  $v(\bar{x}, \bar{x})$* : this generates the well-known effect called ‘loss-aversion,’ and is motivated by the different approach that subjects have to losses as opposed to gains with respect to the reference point.
2. *v is convex below the aspirations level, and concave above it*: this feature, which leads to the so-called ‘diminishing sensitivity,’ is motivated by the fact that the marginal change in gain-loss sensations is greater the closer we get to the reference point. (Notice that  $v$  is strictly concave after  $\bar{x}$ , since there it coincides with  $u$ .)

(See, among others, Kahneman and Tversky (1979), Tversky and Kahneman (1991, 1992) for more discussion on these properties.) While reference-dependent, we assume that  $v$  remains ‘well-behaved:’ the presence of a reference point does not render the function discontinuous or not-monotone (part (1)). Moreover, as opposed to what usually assumed in prospect theory, we posit that the reference point affects the agents’ utility in the area ‘close to’ the aspiration level, but not if ‘far below:’ we posit that there exists a positive  $H$  such that  $v$  is not subject to reference-effects (it coincides with  $u$ ) for levels of  $x$  below  $\bar{x} - H$  (part (2)). The rationale of this restriction is that reference effects are the strongest in the area immediately preceding the reference point, but then they tend to fade, all the way to disappearance, as we get further below. At the same time, we posit that when the reference point is reached subjects should go back to their standard behavior:  $v$  becomes concave and behaves like  $u$  (part (2)).

Because  $v$  depends on the aspirations, but  $u$  does not, then with Assumption 1 and 3 we are imposing that households are reference dependent *only in the second period of their life* – the reference point affects how they value the income later in their life, and their bequest. Conversely, they are not reference dependent when they are young. This approach is motivated by the observation that individuals tend to set objectives

---

<sup>5</sup>By  $\lim_{x \rightarrow \bar{x}^-} v(x, \bar{x})$  we understand the limit of  $v(x, \bar{x})$  as  $x$  approaches  $\bar{x}$  from *below*. (The limit from above is defined analogously.)



for themselves to be accomplished when they have reached a certain age, or for their offsprings to reach, as opposed to for their young age. That is, that the accomplishment of life goals tends to be evaluated only in the later part of life. (A similar approach appears in Mookherjee et al. (2010) and Genicot and Ray (2009).)

Thus far in our analysis we have not imposed any restriction on the origin of agent's aspirations. In this paper we choose to focus on a very specific source for the aspirations: we restrict our attention to the case in which the aspirations level for income in period  $t$  is the *average* income of the society *as a whole* in period  $t$ . (Denote by  $\mu_t$  the distribution of endowments at time  $t$ .)

**Assumption 4.** For  $i \in I$ ,  $t \in T$ ,  $\bar{x}_{i,t} = \int_I x_{i,t} \mu_t(x) di$ .

With Assumption 4 we are ruling out the case in which aspirations are affected by the income of the parents, *à la* Koszegi and Rabin (2009), or of the neighbors, *à la* Mookherjee, Napel, and Ray (2010). Our focus on this special case is motivated by our interest in understanding the effect of this society-wide reference-dependence.

## 2.2 Notion and multiplicity of equilibria

One of the features of the economy described above is that each agent's utility depends on the behavior of the other members of the economy. Since the different agents decides *simultaneously*, the expectations of each agent on the behavior of others will play an essential role: agents are best reacting to what they expect others to do. In line with this, we focus our attention on the following (standard) kind of equilibria.

**Definition 1.** An equilibrium of the economy is a sequence of consumption decisions, bequest decisions, and factor prices

$$e = \{\{c_{it}\}_{i \in L}, \{x_{it+1}\}_{i \in L}, w_t, r_t\}_{t=0}^{\infty}$$

such that factor markets clear and  $(c_{it}, x_{it+1})$  solves (1) for all  $i \in L$  and for all  $t$ .

Definition 1 is a standard notion of equilibrium. Notice that by using this definition together with Assumption 4, we are implicitly assuming that subjects can correctly forecast the behavior of others: in fact, aspirations are defined as the average of the true income distribution in the second period of the agents' life, but affect their behavior already in the first – which implicitly implies that households can correctly forecast it.<sup>6</sup>

It is standard practice to show that one such equilibrium exists for any economy that satisfies Assumptions 1-4. As opposed to the case without aspirations, however, the economy described in Section 2.1 will not admit a unique equilibrium. Rather, equilibria will be 'deeply' non-unique: the presence of aspirations, by rendering the agent's utility

---

<sup>6</sup>Alternatively, we could have defined the aspirations as the expectations of the distribution in the future period, and added the requirement that in equilibrium such expectations are correct and degenerate. The two approaches are clearly identical.

dependent on the behavior of others, introduces obvious coordination issues. To see why, consider the simplest possible case: an economy in which all agents start with the same endowment in the first period. In this simple case, all agents have the same aspirations, and face the same problem: they will then choose the same consumption plan.<sup>7</sup> And since all households have the same consumption, they will all reach their aspirations – they will all invest exactly what needed to reach the average endowment. The problem is, however, that how high this endowment will be depends on the future behavior of the other households. Consider for example some equilibrium in which all households choose some  $a > 0$  in the second period (and each of them knows that this will happen). Then, consider some alternative equilibrium in which, instead, each household knows that all others agents will choose  $a + \epsilon$  (where  $\epsilon$  is small and positive). In this latter case, if one household were to consume only  $a$ , it would fall short of its aspirations: as long as  $\epsilon$  is small it might choose instead to save a little more in the first period so that to reach the rest of the society – and so its aspirations. And since this can be true for all agents, then everybody will in fact choose  $a + \epsilon$  – guaranteeing that this is an equilibrium. Naturally this is true only as long as  $\epsilon$  is ‘small’: there will be some  $\bar{\epsilon}$  large enough such that, even if every agent knew that everyone else will consume  $a + \bar{\epsilon}$ , they’d rather fail to reach their aspirations and consume only  $a$ . And since this will be true for everybody, then the average consumption will be only  $a$  – and thus there cannot be an equilibrium with second-period consumption  $a + \bar{\epsilon}$ , for  $\bar{\epsilon}$  arbitrarily large. In the analysis that follows we will therefore analyze the features of the *set* of possible equilibria of the economy.

Before we proceed, let us briefly add two small remarks, one about some simple features of the growth path and of the optimal individual solution, and one about the role of credit constraints for our results.

**Remark 1.** Since there exists two assets (human and physical capital) and given the assumption on the production function, at every  $t$  we must have  $w_t = 1 + r_t$ . Then the household problem simplifies to the choice of  $I_{t+1}$  to maximize:

$$\max_{I_{t+1}} u(x_{it} - I_{t+1}) + \beta v(w_t I_{t+1}) \quad (2)$$

As customary define  $K_t = \int_L k_{it} di$ ,  $H_t = \int_L e_{it} di$ ,  $\lambda_t = \frac{K_t}{H_t}$  and  $f(\lambda_t) = F(K_t, H_t)/H_t$ . (Notice that by Assumption 2 there are constant returns to scale.) By the arbitrage condition above, and since factor rental prices are equal to the marginal productivity at every  $t$ , we must have

$$f'(\lambda_t) = f(\lambda_t) - f'(\lambda_t)\lambda_t. \quad (3)$$

By Assumption 2 there exists a unique  $\lambda$  which satisfies (3), since the LHS is always decreasing and the right hand side always increasing.

As a result, the economy is in balanced growth path since period zero and the growth rate of the economy is equal to the growth rate of investment. From now on, we use  $w^* = f(\lambda^*) - f'(\lambda^*)\lambda^*$  to indicate the wage rate at every  $t$ .

---

<sup>7</sup>Notice that this requires that subjects cannot make different choices when indifferent. This is proved in Lemma 1 in the Appendix.

**Remark 2.** Since we are mostly interested in the role of aspirations for developing countries, where credit constraints are binding, we have chosen to focus on the case in which access to credit markets is restricted. However, this restriction does not seem to be what is driving our result. In fact, the most important point in our setup is the possibility for households to be in ‘aspiration failure,’ i.e. have a second-period consumption below the aspirations. We will now show that even if we remove any restriction in the access to credit, households might still choose to be in ‘aspirations failure’ – leading to results qualitatively similar to the ones we discuss.

Assume an economy as described above, but with no credit constraints, and notice that the households problem (2) can be re-written as (define  $g$  to be the return on bonds)

$$\max_{I_{t+1}, b_t} u(x_{it} - I_{t+1} + b_{it}) + \beta v(w_t I_{t+1} - g_t b_t). \quad (4)$$

If we compute the FOCs with respect to  $I_{t+1}$  and  $b_t$ , we get:

$$\begin{aligned} u'(x_{it} - I_{t+1} + b_{it}) &= \beta w_t v'(w_t I_{t+1} - g_t b_t) \\ u'(x_{it} - I_{t+1} + b_{it}) &= \beta g v'(w_t I_{t+1} - g_t b_t) \end{aligned} \quad (5)$$

which implies  $g = w_t = w^*$ . The problem is then determined in the difference  $I - b$ . It is then easy to see that, if  $w^*(I - b) = \bar{x}_{it+1}$  and if

$$u'(x_{it} - \bar{x}_{it+1}/w^*) + \lim_{x' \rightarrow \bar{x}_{it+1}^-} \beta w^* v'(x') > 0 \quad (6)$$

then the optimal choice of the household is to be in aspirations failure, as sought.

### 2.3 Properties of the optimal behavior

Because of the convexity and non-differentiability of the second-period utility function, the behavior of households in this economy will be different from that in the standard model. To better express its features, let us define the optimal choice of an household in the second period as a function of the initial endowment and aspiration level:<sup>8</sup> define  $\phi : \mathbb{R}_{++} \times \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$  such that  $x_{i,t+1} = \phi(e, \bar{x})$  is an optimal solution of (1) for an household  $i$  with initial endowment  $e$  and aspirations  $\bar{x}$ . Then, the following must hold.

**Proposition 1.** *The following holds for all  $\bar{x}$ :*

1. *there exists  $\gamma \in \mathbb{R}$ ,  $\gamma > 0$  such that for all  $e \in \mathbb{R}_{++}$ ,  $\phi(e, \bar{x}) \notin (\bar{x}_t - \gamma; \bar{x}_t)$ ;*
2. *there exist some  $\underline{e}, \bar{e} \in \mathbb{R}_{++}$ ,  $\underline{e} < \bar{e}$  such that  $\phi(e, \bar{x}) = \bar{x}$  for all  $e \in (\underline{e}, \bar{e})$ .*

---

<sup>8</sup>To be precise, this map should also depend on the other elements of the economy, like  $r_t$  and  $w_t$ . We omit them from the notation for simplicity.

Proposition 1 shows two features of the optimal solution of the household problem. Part (1) shows that there is an interval (of positive measure) of second period income, right before the aspirations level  $\bar{x}$ , that will *not* be chosen by any household with aspirations  $\bar{x}$ , irrespectively of their initial endowment. Put differently, no household will choose a second-period income too close below the aspirations: either they reach their aspirations, or they fall short of them of a non-trivial amount. This is naturally due to the fact that  $v$  is convex in an interval that ends in  $\bar{x}$ . As we shall see, this has important consequences on the aggregate behavior of the economy.

Part (2) of Proposition 1 shows another features of the optimal behavior: there is a non-zero interval of initial endowments such that the optimal choice for households with those endowments is the same and equal to the aspirations level. That is, many different households, with many different initial endowments, will choose to consume *exactly* same amount in the second period, equal to the aspirations.

### 3. Aspirations, Distribution and Frustration in the Long Run

#### 3.1 Distribution in the long run

We now turn to study the distribution of our economy in the long run. We start with two (standard) definitions: polarized distribution and perfect equality.

**Definition 2.** A distribution  $\mu$  on  $\mathbb{R}_+$  is of perfect equality if  $\mu(x) = 1$  for some  $x \in \mathbb{R}_+$ .

**Definition 3.** A distribution  $\mu$  on  $\mathbb{R}_+$  is polarized around  $a$  if there exist  $x_1, x_2 \in \mathbb{R}_+$ ,  $x_1 < a \leq x_2$  such that:

1.  $\mu([0, x_1)) > 0$  and  $\mu([x_2, +\infty)) > 0$ ;
2.  $\mu(x) = 0$  for all  $x \in [x_1, x_2]$ .

A distribution is of perfect equality if it is degenerate. By contrast, a distribution is polarized if it has a “gap” of positive measure. In particular, a distribution is polarized around its mean if it has this “gap” in the middle, to divide those above the mean on the one side, from the those below the mean on the other. It turns out that, in the presence of aspirations, these two types of distributions are enough to describe the long run behavior of any the economy that satisfy our assumptions. (By  $\mu_t^e$  we denote the distribution of endowment in equilibrium  $e$  and period  $t$ .)

**Theorem 1.** *Consider an economy as described above that satisfies Assumptions 1-4, and some equilibrium  $e$  of this economy. Then there exists some  $T \in \mathbb{N}$  such that for all  $t > T$ , one of the following must hold:*

- (a)  $\mu_t^e$  is polarized around  $\mathbb{E}[\mu_t]$ ;
- (b)  $\mu_t^e$  is of perfect equality.

Theorem 1 shows that the long run distribution of any equilibrium of any economy that satisfy our assumptions can be of only two kinds: either polarized, or of perfect equality. No other distribution is possible – in particular, no continuous and non-degenerate distributions are possible in the long run. Notice, moreover, that this is not an asymptotic result, or a property of the steady state. Rather, Theorem 1 shows that one of these two types of distribution will be reached in *finite time* (at time  $T$ ), and will be stable: after that time the economy will be, and forever remain, either polarized or of perfect equality.

To analyze the implication of the theorem, recall that in the standard case in which  $u$  and  $v$  are identical and concave, an economy that started with a continuous and non-degenerate distribution of initial endowments could easily have an equilibrium in which the distribution is continuous and non-degenerate in every period: for example, if  $u$  exhibits constant relative risk aversion (CRRA), then any *distribution is possible in the long run* if  $u = v$  and both are concave.<sup>9</sup> By contrast, Theorem 1 shows that the presence of aspirations has a strong impact on the type of paths that are attainable in equilibrium: no matter what the initial distribution is, and no matter how small the impact of aspirations on utilities is, a continuous non-degenerate distribution is no longer possible in the long run. Either the poor get “left behind,” failing to attain their aspirations, and the society is polarized between those that attain their aspirations and those that fail to. Or, everybody catches up with their aspirations – but this can happen only if that the distribution of endowments is of perfect equality.

### 3.2 Origin of the long run distribution

Theorem 1 shows that in any equilibrium the long run distribution of endowments can be of only two kinds, but it does not provide conditions as to when each of these two cases take place. This is the content of the following proposition.

**Proposition 2.** *Consider an economy as described above that satisfies Assumptions 1-4 with some initial distribution of the endowment  $\mu_0$ . Then each of the following must hold:*

1. *if  $\mu_0$  is not degenerate, then there exists some equilibrium  $e$  and some  $T \in \mathbb{N}$ , such that for all  $t > T$ ,  $\mu_t^e$  is polarized around  $\mathbb{E}[\mu_t^e]$ ;*
2. *there exists some  $\delta \in \mathbb{R}$ ,  $\delta > 0$ , such that, if  $\frac{\inf_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta$ , then there exists an equilibrium  $e$ , and some  $T \in \mathbb{N}$ , such that for all  $t > T$ ,  $\mu_t^e$  is of perfect equality.*

Proposition 2 shows that the distribution that the economy will assume in the long run depends both on the initial conditions ( $\mu_0$ ), and also on the equilibrium we are looking

---

<sup>9</sup>To see why, notice that in an economy like ours but in which  $u = v$  and both are strictly concave and exhibit CRRA, the distribution of endowments is constant over time. (This is proved in Lemma 2 in the Appendix.) But this implies that the initial distribution is also the long run distribution, which means that any long run distribution is possible.

at. In particular, part (1) shows that any economy admits a long run equilibrium that evolves into a polarized distribution, no matter what the initial distribution is (as long as it is not degenerate). To get an intuition of why this is the case, notice that we can always construct an equilibrium in which every member of the society with an endowment equal or above the mean is expected to increase her consumption by at least some  $b$ , where  $b$  is defined as the maximum amount that the household with an endowment exactly equal to the mean is willing to increase her consumption to if she knew that the mean will increase by  $b$ . In this equilibrium everyone with an endowment above or equal to the mean will “keep up,” while the rest will not, generating polarization. If this is true in every period this polarization will never disappear, leading to part (1) of the proposition.

On the other hand, part (2) of Proposition 2 shows that, if the initial distribution is sufficiently “condensed,” i.e. the ratio between the minimum and the average income is high ( $\frac{\inf_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta$ ), then this economy will *also* admit an equilibrium which evolves into perfect equality in the long run. The reason is, again, very simple: if the poor are not too much behind the average, as time goes by they will have time to catch up – leading to perfect equality.

## 4. Growth

### 4.1 Comparative Growth

We now turn to investigate the relation between inequality and growth in the presence of reference-effects. It is well known that with standard preference the relation between inequality and growth is complex (see Aghion et al. (1999)). In our simple setup, however, if we remove any reference-effect (i.e. we posit  $u = v$  and both concave), and we focus on CRRA utility, we get a unique growth rate regardless of the initial distribution.<sup>10</sup> In this section we will show that in our model with reference-dependence the initial distribution will always affect the growth of the economy, even if the utility is CRRA. In particular, we shall compare how the average income changes over time depending on the initial distribution of the society, and on the equilibrium that we are looking at.

To better express these comparisons, let us introduce a few simple concepts. We begin by saying that an equilibrium *dominates* another if the average endowment of the first is above the average endowment of the other for every period starting from some period  $T$ : that is, if there is a point in which the average income of the first surpasses, and stays above, that of the latter. We shall use the symbol  $\succeq$  ( $\succ$ ) to express this (strict) relation. **Definition 4.** For any equilibrium  $e$  of some economy  $E$ , and equilibrium  $e'$  of some economy  $E'$ , we say  $e \succeq e'$  if there exists some  $T \in \mathbb{N}$  such that for all  $t \geq T$  we have

$$\int_x x \mu_t^e(x) dx \geq \int_x x \mu_t^{e'}(x) dx.$$

---

<sup>10</sup>This is naturally not necessarily true for other utility functions. See Lemma 2 in the Appendix.

We say that  $e \succ e'$  if there exists some  $T \in \mathbb{N}$  such that the inequality above is strict for all  $t \geq T$ .

Notice that the comparison  $\succeq$  could also be seen also as a comparison of the average growth rate of two economies that start from the same initial average endowment. (If we have two economies  $E$  and  $E'$  with the same average endowment, and equilibria  $e$  of  $E$  and  $e'$  of  $E'$  such that  $e \succ e'$ , then in the long run not only the endowment, but also the average growth rate of  $e$  is above that of  $e'$ .)

While  $\succeq$  compares two specific equilibria, potentially of different economies, we now introduce a comparative notion for *all* the equilibria of two economies. We define three notions:  $\triangleright$ ,  $\triangleright_{\max}$ ,  $\triangleright_{\min}$ , to represent, respectively, full dominance, higher “best” equilibrium, and higher “worst” equilibrium. We start with “full dominance.”

**Definition 5.** For any economies  $E$  and  $E'$  we say that  $E \triangleright E'$  if for all equilibria  $e$  of  $E$ , and  $e'$  of  $E'$ , we have  $e \succ e'$ . We say  $E \triangleright_{\max} E'$  if for all equilibria  $e$  of  $E$ , and  $e'$  of  $E'$ , we have  $e \succeq e'$ .

The idea behind Definition 5 is that if one economy  $E$  “dominates” another economy  $E'$  ( $E \triangleright E'$ ), then it means that every equilibrium of  $E$  dominates (in the sense of  $\succ$  above) every equilibrium of  $E'$ . This notion, however, is very demanding, since it requires that even the worst of the equilibria of  $E$  strictly dominates the best of the equilibria of  $E'$ . Two weaker notions would compare the ‘best’ equilibrium of each economy, or the ‘worst’ one. This is what we do in the definition of  $\triangleright_{\max}$  and  $\triangleright_{\min}$  below.

**Definition 6.** For any economies  $E$  and  $E'$  we say that  $E \triangleright_{\max} E'$  if there exists some  $e$  of  $E$  such that  $e \succ e'$  for all  $e'$  of  $E'$ . We say  $E \triangleright_{\min} E'$  if there exists some  $e$  of  $E$  such that  $e \succeq e'$  for all  $e'$  of  $E'$ .

**Definition 7.** For any economies  $E$  and  $E'$  we say that  $E \triangleright_{\min} E'$  if there exists some  $e'$  of  $E'$  such that  $e \succ e'$  for all  $e$  of  $E$ . We say that  $E \triangleright_{\max} E'$  if there exists some  $e'$  of  $E'$  such that  $e \succeq e'$  for all  $e$  of  $E$ .

We say that an economy  $\triangleright_{\max}$ -dominates another if its best equilibrium dominates (in the sense of  $\succ$ ) *any* equilibrium of the other economy. Conversely, we say that an economy  $\triangleright_{\min}$ -dominates another if even its worst equilibrium dominates (again in the sense of  $\succ$ ) *some* equilibrium of the other.

Finally, in order to better express our results on growth, we compare the behavior of four different types of economies, each of which is assumed to have the same average initial endowment.

**Definition 8.** Denote by  $\mathcal{E}_0$  the set of economies that satisfy Assumptions 1, 2,  $v = u$  and such that  $\mu_o$  is of perfect equality.  $\mathcal{E}_1$  is the set of economies that satisfy Assumption 1-4 and such that  $\mu_o$  is of perfect equality.  $\mathcal{E}_2$  is the set of economies that satisfy Assumptions 1-4, in which initial distribution is not of perfect equality, but there admits an equilibrium  $e$  and some  $T \in \mathbb{N}$  such that for all  $t \geq T$ ,  $\mu_t^e$  is of perfect equality.  $\mathcal{E}_3$  is the set of economies that satisfy Assumptions 1-4, in which initial distribution is not of perfect equality, and that admits no equilibrium  $e'$  and some  $T \in \mathbb{N}$  such that for all  $t \geq T$ ,  $\mu_t^{e'}$  is of perfect equality.

Economies in  $\mathcal{E}_0$  is a standard economy with a representative agent who has a standard concave utility function with no reference-effects. Economies in  $\mathcal{E}_1$ ,  $\mathcal{E}_2$ , and  $\mathcal{E}_3$ , instead, have households with aspirations as modeled in Assumptions 1-4. They differ from each other because of the initial distribution: economies in  $\mathcal{E}_1$  have an initial distribution with perfect equality; those in  $\mathcal{E}_2$  do not, but admits at least one equilibrium in which the wealth distribution converges to perfect equality (in finite time); those in  $\mathcal{E}_3$ , instead starts with a distribution with some inequality, and admit no equilibria in which the inequality disappears. From Proposition 2 we know that we can think of economies in  $\mathcal{E}_2$  as those in which we have some initial inequality, but in which the wealth distribution is very ‘condensed’ – no household has an initial endowment too far below the mean endowment. By contrast, an economy in  $\mathcal{E}_3$  would be an economy in which the initial distribution has a more acute inequality: there are households with initial endowments far from the mean.

#### 4.2 Growth in the ‘best’ equilibria

We are then ready for the first result on growth. Let us start by looking at the ‘best’ equilibria – those in which growth is the highest.

**Theorem 2.** *Consider four economies  $E_0 \in \mathcal{E}_0$ ,  $E_1 \in \mathcal{E}_1$ ,  $E_{2,3} \in \mathcal{E}_2 \cup \mathcal{E}_3$  that have the same initial average endowment. Then:*

$$E_1 \succ_{\max} E_{2,3} \quad \text{and} \quad E_1 \succ_{\max} E_0.$$

*If  $u$  exhibit CRRA, then:*

$$E_1 \succ_{\max} E_{2,3} \succeq_{\max} E_0.$$

Theorem 2 shows that the presence of aspirations could have a strong impact on growth depending on the initial distribution. First of all, it shows that there exists an equilibrium of  $E_1$  that dominates *all* other equilibria of *all* other economies: the equilibrium with the (strictly) highest growth amongst all is one of an economy with aspirations and with an initial distribution of perfect equality. The intuition is simple: with aspirations and perfect equality, agents could “push” each other into consuming more, and since the distribution is of perfect equality, then by doing this there is no subject that is “left behind” – leading to the highest growth. In this equilibrium, growth will be strictly higher also of any equilibrium of the economy with no aspirations ( $E_0$ ): the reason is, without aspirations we loose the mechanism of “pushing each other” which makes the growth so high in the case of  $E_1$ . This growth will also be higher than the growth with aspirations and initial inequality: in this case, the wealthy cannot expect the society to increase the income too much, since they know that the poor would not follow, and therefore they won’t increase their own income as much as they do in  $E_1$ .

If we further assume that  $u$  exhibits constant relative risk aversion, then we obtain the full, transitive rank:  $E_1 \succ_{\max} E_{2,3} \succeq_{\max} E_0$ . That is, if we look at the best equilibria, we have that the highest growth is found with aspirations and perfect equality, then



with aspirations and initial inequality, and finally the economy with no aspirations. This should be compared with what happens when there are no aspirations, and both  $u$  and  $v$  are identical, concave, and CRRA: it is well known (we show it again in Lemma 2 in the Appendix) that in this case the growth rate is constant and independent of the initial distribution. (Notice that this implies that the ranking above is true also for some  $E'_0$  which, like  $E_0$ , has no aspirations, but that has an initial distribution which is not necessarily of perfect equality.) That is: the presence of aspirations is always positive, but it renders growth dependent of the initial distribution, where the presence of initial inequality is actually harmful for growth.

This discussion shows a feature of society-influenced reference-dependence. On the one hand, it induces households to ‘push each other,’ generating growth. On the other hand, this mechanism works only insofar as households are not ‘too far’ from each other – if they are, this mechanism would not work.

#### 4.3 Comparative growth in all equilibria

Theorem 2 analyzes the ranking only for the ‘best’ equilibria. It turns out that things can be quite different in other equilibria.

**Theorem 3.** *Consider four economies  $E_0 \in \mathcal{E}_0$ ,  $E_1 \in \mathcal{E}_1$ ,  $E_2 \in \mathcal{E}_2$ , and  $E_3 \in \mathcal{E}_3$  that have the same average initial endowment. Then the following holds:*

1.  $E_1 \supseteq E_0$ ;
2. *for any equilibrium  $e$  of  $E_0$ , there exists some equilibrium  $e'$  of  $E_1$  such that  $\mu_t^e = \mu_t^{e'}$ .*

*Moreover, if  $u$  exhibits CRRA, then:*

4.  $E_2 \supset E_0$ ;
5.  $E_2 \supset_{\min} E_1$ ;
6.  $E_3 \supseteq E_0$ ;
7.  $E_3 \supseteq_{\min} E_1$ .

Theorem 3 considers other equilibria besides those with highest growth. First, it shows that the presence of aspirations never reduces growth: every equilibrium of  $E_1$  must grow at least as much as any equilibrium of  $E_0$  ( $E_1 \supseteq E_0$ ); and if  $u$  is CRRA, then also any equilibria of  $E_2$  and  $E_3$  grow more than that  $E_0$  ( $E_2 \supset E_0$  and  $E_3 \supseteq E_0$ ). In fact, equilibria in  $E_2$  *strictly dominate* all equilibria in  $E_0$ . At the same time, however, the presence of aspirations is not sufficient to have a higher growth: for example, there are equilibria of  $E_1$  in which the wealth distribution coincides *in every period* with that of the (unique) equilibrium of  $E_0$ .

Also in the comparison between  $E_1$ ,  $E_2$ , and  $E_3$ , the ranking in Theorem 3 is quite different from the one in Theorem 2. While in the latter we have seen that the economy with the highest growth in the best equilibrium is  $E_1$ , Theorem 3 shows that, if  $u$  is CRRA, then a society with a small amount of initial inequality ( $E_2$ ) has a lowest-growth equilibrium that grows *strictly* more than the lowest-growth equilibrium of a society with initial perfect equality ( $E_1$ ): we have  $E_2 \triangleright_{\min} E_1$ . Moreover, we also have that  $E_2 \triangleright E_0$ : the presence of a little bit of inequality renders minimal growth *strictly higher* also of the case with no reference-effects. The intuition is that, with aspirations, the households that are right below the average income might choose to ‘push up,’ and reach their aspirations – generating growth. And since this cannot happen with perfect equality or with no aspirations, then we have that the minimal growth of an economy of type  $E_2$  must lie strictly above that of an economy of type  $E_2$  or  $E_0$ . A similar argument suggests why we have  $E_3 \succeq_{\min} E_0$ , and  $E_3 \succeq_{\min} E_1$ . (Here the inequality is weak,  $\succeq$ , since there could be no subjects who ‘push up’ of the kind described above.<sup>11</sup>)

The results of Theorems 2 and 3 can then be summarized as follows: the presence of aspirations can only increase the growth rate with respect to the case of no aspirations. Depending on the initial distribution of the economy, the growth rate will be strictly higher (as is the case if the initial range is small enough), or identical at least for some equilibria (as is the case for initial perfect equality). If we compare the growth rates between economies with aspirations but different initial distributions, we find that the results depend strictly on the equilibrium we are looking at: economies that have an initial distribution with perfect equality have equilibria with a growth rate strictly higher than any other economy; at the same time, they also have minimal equilibria that are worse than the minimal equilibria of any other economy – strictly worse than the minimal equilibria of  $E_2$ . A graphical intuition of the results appears in Figure 4.3, which represent the set of average growth rates for all equilibria of each type of economy. (Notice that both the highest and the lowest growth rate for  $E_3$  could be either above or below those of  $E_2$ , albeit always (weakly) above that of  $E_0$ . To represent this possibility, we Figure 4.3 contains the lighter shade area for  $E_3$ .)

## 5. Conclusion

In this paper we study a warm glove OLG model in which agents’ utility on bequest depends on the the average income of the rest of society, which will act as a reference-point. In line with this interpretation, we model this reference-dependence using the standard prospect theory form. We argue that this leads to multiple equilibria. We show that in any of these equilibria the wealth distribution will converge (in finite time) to be either polarized or of perfect equality. We then turn to study the growth rate of different societies, and show that a society that starts from perfect equality is the one that has the equilibrium that grows the most, followed by a society with initial inequality. At the same time, if we look at equilibria with the lowest growth, then the society that grows

---

<sup>11</sup>That is, there could be no household with an endowment below but close to the average one.

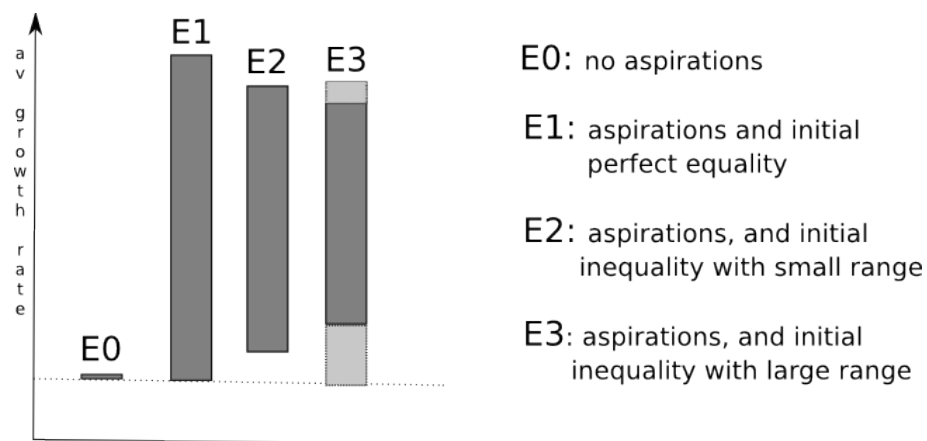


FIGURE 1 Average growth rates for different equilibria of economies  $E_0$ ,  $E_1$ ,  $E_2$ ,  $E_3$  when  $u$  is CRRA

the most is the one with a small amount of initial inequality. Finally, we show that any society with reference-effects, no matter what the initial distribution is, grows more than any society without aspirations.

# Appendix

## Appendix A. Preliminary Results

**Lemma 1.** *For all equilibria  $e$ , if  $\mu_T^e$  is of perfect equality, then  $\mu_t^e$  is of perfect equality  $\forall t \geq T$ .*

*Proof.* Assume that  $\mu_T$  is of perfect equality. We claim that  $x_{i,T+1} = \mathbb{E}[\mu_{T+1}]$  for all  $i \in L$ . By means of contradiction, assume instead that there exists  $i \in L$  such that  $x_{i,T+1} \neq \mathbb{E}[\mu_{T+1}]$ . If we have  $x_{i,T+1} > \mathbb{E}[\mu_{T+1}]$ , there must also exist some  $j \in I$  such that  $\mathbb{E}[\mu_{T+1}] > x_{j,T+1}$ , by definition of  $\mathbb{E}[\mu_{T+1}]$ . Assume then WLOG that we have  $x_{i,T+1} > \mathbb{E}[\mu_{T+1}] > x_{j,T+1}$  for some  $i, j \in L$ . Notice that for both  $i$  and  $j$  we must have that the optimal solution meets the FOCs, i.e.  $u'(x_T - x_{i,T+1}/w^*) = \beta w^* v'(x_{i,T+1})$  and  $u'(x_T - x_{j,T+1}/w^*) = \beta w^* v'(x_{j,T+1})$ . By Assumption 1 and 3, for  $x_{i,T+1} > x_{j,T+1}$  we have that  $\beta v'(x_{j,T+1}) < \beta w^* v'(x_{i,T+1})$  which implies that  $u'(\cdot)$  is increasing between  $x_T - x_{j,T+1}$  and  $x_T - x_{i,T+1}$  which violates Assumption 1, which is a contradiction.  $\square$

**Lemma 2.** *If  $u = v$  and  $u$  is CRRA, the growth rate of the economy is unique and invariant. At the same time, there exists a strictly concave  $u$  such that this is not true.*

*Proof.* Consider the first order condition for (2) imposing  $u(\cdot) = v(\cdot)$  (internal solution is guarantee by Assumption 1) and substitute  $x_{it+1} = \psi_{it}x_{it}$ , we get:

$$-u' \left( x_{it} \left( 1 - \frac{\psi_{it}}{w^*} \right) \right) + \beta w^* u'(x_{it}\psi_{it}) = 0 \quad (7)$$

applying the implicit function theorem we get:

$$\frac{d\psi_{it}}{dx_{it}} = - \frac{\left( 1 - \frac{\psi_{it}}{w^*} \right) u'' \left( x_{it} \left( 1 - \frac{\psi_{it}}{w^*} \right) \right) + \beta w^* u''(x_{it}\psi_{it})\psi_{it}}{\left( \frac{x_{it}}{w^*} \right) u'' \left( x_{it} \left( 1 - \frac{\psi_{it}}{w^*} \right) \right) + \beta w^* u''(x_{it}\psi_{it})x_{it}} \quad (8)$$

where the sign is not *a priori* guaranteed and is clearly dependent on the expression at numerator.

Substituting for  $u(x) = \frac{c^{1-\theta}}{1-\theta}$  it can be easily checked that the growth rate is unique and invariant to the distribution.  $\square$

## Appendix B. Proofs

*Proof of Proposition 1*

To prove both points consider the household problem 1, and notice that the FOC are

$$\lim_{x' \rightarrow \bar{x}_{it+1}^-} \beta w^* v'(x') \geq u' \left( x_{it} - \frac{x_{it+1}}{w^*} \right) \geq \lim_{x' \rightarrow \bar{x}_{it+1}^+} \beta w^* v'(x')$$

Notice that if no  $x \in (0, \bar{x}_{it+1})$  satisfy them, the claim is trivially true. Otherwise, define  $x^\alpha$  the highest  $x \in (0, \bar{x}_{it+1})$  that satisfies the FOC. (The existence of  $x^\alpha$  is guaranteed by standard arguments.) By Assumption 3, for all  $\epsilon > 0$  we must have  $-u'(x - x^\alpha/w^* - \epsilon/w^*) + \beta w^* v'(x^\alpha + \epsilon) > 0$  and

$-u'(x - x^\alpha/w^* + \epsilon/w^*) + \beta w^* v'(x^\alpha - \epsilon) < 0$ , which implies that  $x^\alpha$  cannot be a maximum since it fails the second order conditions (recall that since  $x < \bar{x}_{it+1}$ , the function is differentiable at  $x^\alpha$ ). In turns, this implies that  $\bar{x}_{it+1}$  is an optimal solution for all initial endowments  $x$  such that  $u'(x - \bar{x}_{it+1}/w^*) \in [\lim_{x' \rightarrow \bar{x}_{it+1}^+} \beta w^* v'(x'), \lim_{x' \rightarrow \bar{x}_{it+1}^-} \beta w^* v'(x')]$ .

*Q.E.D.*

*Proof of Theorem 1*

We start by noticing that, from period 1 onwards, the distribution of endowments must be either polarized, or of perfect equality. To see why, consider any initial distribution  $\mu_0$ , and notice that by Proposition 1 we know that there exists some  $\gamma > 0$  such that  $\mu((\mathbb{E}[\mu_1]_1 - \gamma, \mathbb{E}[\mu_1]_1)) = 0$ . This implies that, if  $\mu_1([0, \mathbb{E}[\mu_1])) > 0$ , then  $\mu_1([0, \mathbb{E}[\mu_1] - \gamma)) > 0$ . Notice also that, by construction, we must have that  $\mu_1([\mathbb{E}[\mu_1], +\infty)) > 0$ . Therefore, if  $\mu_1([0, \mathbb{E}[\mu_1])) > 0$ , then the distribution is polarized around  $\mathbb{E}[\mu_1]$ . Conversely, if  $\mu_1([0, \mathbb{E}[\mu_1])) = 0$ , then we must have  $\mu_1(\{\mathbb{E}[\mu_1]\}) = 1$  (every distribution with a support above its average must be degenerate). We have therefore proved that  $\mu_1$  can be either polarized around  $\mathbb{E}[\mu_1]$ , or of perfect equality. An identical argument shows that the same would hold true for all  $\mu_t$  for all  $t$ .

We are only left to show that there exists some  $\bar{T}$  from which the distribution is either of perfect equality, or polarized. Notice that, if  $\mu_t$  is of perfect equality, so will be  $\mu_{t+1}$ . Therefore if there exists some  $T$  such that  $\mu_T$  is of perfect equality, then we can set  $\bar{T} = T$ . Otherwise, if such  $T$  does not exist, we can set  $\bar{T} = 1$ , since we have proved that  $\mu_1$  is polarized if it is not of perfect equality. *Q.E.D.*

*Proof of Proposition 2*

1) Consider some non-degenerate  $\mu_t$ , and notice that if  $\mu_t$  is not continuous at its mean, then it must be polarized. Otherwise, consider some  $\mu_0$  which is continuous around its mean, and consider the equilibrium in which each  $i$  at time  $t$  chooses  $\max_{I'_{t+1}} \{I'_{t+1} \in \arg \max_{I_{t+1}} u(x_0 - I_{t+1}) + \beta v(w_t I_{t+1} | w_t I_{t+1}^*)\}$ . Then there exists some  $x_{it}$  such that  $\lim_{I' \rightarrow \bar{I}^-} \beta w^* v'(w^* I', w^* I') = u'(x_{it} - \bar{I})$ . Notice that we must have that  $x_{it}$  is in the interior of the support of  $\mu_t$ , otherwise this would violate Assumption 4 (aspirations are equal the average income).

This  $x_{it}$  should not stay on the lower bound of the support because it will violates Assumption 4. But then  $\forall x_{jt} < x_{it}$ , household  $j$  will not reach her aspirations, and by Proposition 1 the distribution will be polarized. Since  $t$  has been chosen arbitrarily, this proves the first part of the proposition.

2) By Proposition 1, we know that there exists an interval  $S \subseteq \mathbb{R}_+$  such that  $\forall x' \in S, \bar{x} = \phi(x', \bar{x})$ . This clearly implies that if  $\text{supp}(\mu_0) \subseteq S$ , then  $\mu_1$  is of perfect equality. By Lemma 1 we also know that it will remain in perfect equality for all  $t$ . We can define  $\delta > 0$  implicitly as any  $\delta > 0$  such that

$$\frac{\min_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta \Rightarrow \text{Supp}(\mu_0) \subseteq S.$$

*Q.E.D.*

*Proof of Theorem 2 and 3*

Consider four economies  $E_0 \in \mathcal{E}_O$ ,  $E_1 \in \mathcal{E}_1$ ,  $E_2 \in \mathcal{E}_2$ , and  $E_3 \in \mathcal{E}_3$  that have the same average initial endowment. Call  $e_0$  the unique equilibrium of  $E_0$ , and  $x_t^{e_0}, \dots$ , the average endowments in  $e_0$  at period

$t$ . Then, the following holds.

**Claim 1.** For all  $t > 0$ , there exists a compact, convex, and positive-measure set  $S_t$  such that for all equilibria  $e$  of  $E_1$ ,  $\mathbb{E}[\mu_t^e] \in S_t$ . Moreover, for all  $t > 0$  we have  $x_t^{e_0} \in S_t$  and  $x_t^{e_0} \leq x'_t$ ,  $\forall x' \in S_t$ .

*Proof.* Notice first that in any equilibrium of  $E_1$  the distribution must remain of perfect equality in all periods by Lemma 1. Then, in every period every household must choose a consumption exactly equal to her aspirations, which implies that in all equilibria we must have  $x_{i,t} = \phi(x_{i,t-1}, x_{i,t})$  for all  $i \in L$ , for all  $t$ . In turns, this means that we have  $\bar{x}_{t+1}$  such that  $\lim_{x' \rightarrow \bar{x}_{t+1}^-} \beta w^* v'(x') \geq u'(x - \bar{x}_{t+1}/w^*) \geq \lim_{x' \rightarrow \bar{x}_{t+1}^+} \beta w^* v'(x')$ . Since the two limits are finite, by Assumption 1  $E_1$  has a set of equilibrium solutions such that at every  $t > 0$  the optimal  $x_t^*$  belong to a compact convex set  $S_t$ . In turns, this implies that  $S_t$  is a set of positive measure for all  $t$ .

Notice that  $x_t^{e_0}$  satisfies first order conditions as an interior point by Assumption 1. Consider now  $x'_t \in S_t$ . Assume that  $x'_t < x_t^{e_0}$ . By Assumption 4, the aspirations level should be equal to  $x'_t$ . Since by Assumption 3 the representative agents in  $E^0$  and  $E^1$  have the same utility outside an interval the highest point of which is the aspirations, then between  $x'_t$  and  $x_t^{e_0}$  the  $u'(\cdot)$  must be either constant or increasing and decreasing. But this contradicts Assumption 1.

We are left to show that  $x_t^{e_0} \in S_t$ . Assume by contradiction that the  $x_t^{\min}$  defined as  $u'(x_{t-1} - x_t^{\min}/w^*) = \lim_{x \rightarrow x_t^{\min}} \beta w^* v'(x)$  is such that  $x_t^{\min} > x_t^{e_0}$ . This implies  $u'(x_{t-1} - x_t^{e_0}/w^*) < \beta w^* v'(x_t^{e_0})$  but since  $u' = v'$  above the aspiration level,  $x_t^{\min}$  is a stationary point of the problem without aspirations, which implies a violation of Assumption 1.  $\square$

Notice that Claim 1 implies  $E_1 \supseteq_{\max} E_0$ . This, together with the observation above that  $S_t$  has positive measure for all  $t$ , implies that  $E_1 \supset_{\max} E_0$ . (The reason is,  $E_1$  has multiple equilibria with different growth, and all of them have a growth weakly above  $E_0$ ; but then, there must exist an equilibrium with a growth strictly above  $E_0$ .) In turns, Claim 1 also proves points (1) and (2) of Theorem 3.

We will now show that  $E_1 \supset_{\max} E$  for all  $E \in \mathcal{E}_2 \cup \mathcal{E}_3$ . Denote by  $e_1$  and  $e$  the equilibria of maximum expansion of  $E_1$  and  $E$ , respectively. Also, denote  $e_t$  the equilibrium of maximum expansion of an economy which in period  $t$  has the same average endowment as  $e$ , but has perfect equality at time  $t$ , i.e. an economy of type  $\mathcal{E}_1$  such that  $\mu_t^{e_t}(\mathbb{E}[\mu_t^e]) = 1$ . We will first of all show that  $e_t$  has an average consumption that grows strictly more than  $e$  between time  $t$  and time  $t+1$  for all  $t$  such that  $\mu_t^e$  is not of perfect equality. (If  $\mu_t^e$  is of perfect equality then  $e$  and  $e_t$  coincide.). To see why, consider first the case in which  $\mu_{t+1}^e$  is of perfect equality. By definition of maximum rate of expansion,  $\mathbb{E}[\mu_{t+1}^{e_t}] = \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_{t+1}^{e_t}] - y/w^*) + \beta v(y, y)\}$ . We claim that  $\mathbb{E}[\mu_{t+1}^e] \leq \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_{t+1}^e] - y/w^*) + \beta v(y, y)\}$  where the inequality is strict if  $\mu_t^e$  is not of perfect equality. If this were not the case, there would exist  $i \in L$  such that  $x_{i,t+1}^e = \max_{x'} \{x' \in \arg \max_y u(x_{i,t} - y/w^*) + \beta v(y, y)\}$ . This, however, would mean that then the distribution is not of perfect equality, which is a contradiction. Then  $x_{t+1}^e < \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_{t+1}^e] - y/w^*) + \beta v(y, y)\}$ , proving that if  $\mu_{t+1}^e$  is of perfect equality then  $e_t$  has an average consumption that grows strictly more than  $e$ .

Consider now the case in which  $\mu_{t+1}^e$  is not of perfect equality.

Fix some  $t \geq 0$ , and consider  $\bar{x} \in \mathbb{R}_{++}$  such that  $\mathbb{E}[\mu_{t+1}^e] = \max_{x'} \{x' \in \arg \max_y u(\bar{x} - y/w^*) + \beta v(y, y)\}$ . This  $\bar{x}$  should be strictly greater than the lower bound of the support and strictly less than the higher bound by definition of polarized distribution.

**Claim 2.**  $\bar{x} < \mathbb{E}[\mu_t^e]$

*Proof.* By contradiction, assume  $\bar{x} \geq \mathbb{E}[\mu_t^e]$ . Then by definition of  $\bar{x}$ , all the income strictly lower should determine failure of aspirations, i.e.  $\forall j \in L$  such that  $x_{j,t} < \bar{x}$  we have  $x_{j,t+1} < \mathbb{E}[\mu_{t+1}^e]$ . Consider now

$k \in L$  such that  $x_{kt} > \bar{x}$ , and notice that either  $x_{k,t+1} = \mathbb{E}[\mu_{t+1}^e]$ , or  $x_{k,t+1} = \gamma(x_{kt})x_{kt} > \mathbb{E}[\mu_{t+1}^e]$ , where the growth rate  $\gamma(x_{kt})$  is defined by  $-u' \left( x - \frac{x\gamma(x)}{w^*} \right) + \beta w^* v'(x\gamma(x)|x\gamma') = 0$ . (Notice that this last expression is twice differentiable.) Then, notice that we must have

$$\frac{d\gamma(x)}{dx} = \frac{\beta w^* \frac{dv'(x\gamma(x)|x\gamma')}{dx\gamma'} \gamma'}{-\left[ -u'' \left( x - \frac{x\gamma(x)}{w^*} \right) \frac{x}{w^*} + \beta w^* v'(x\gamma(x)|x\gamma') x \right]} \leq 0. \quad (9)$$

Call  $\gamma^* = \frac{E[\mu_{t+1}^e]}{\bar{x}}$ , we have:

$$\begin{aligned} \mathbb{E}[\mu_{t+1}^e] &= \int_{j \in L \mid x_{jt} < \bar{x}} x_{jt+1} d\mu_{t+1} + \int_{k \in L \mid x_{kt} \geq \bar{x}} x_{kt+1} d\mu_{t+1} = \\ &< \gamma^* \int_{j \in L \mid x_{jt} < \bar{x}} x_{jt} d\mu_t + \gamma^* \int_{k \in L \mid x_{kt} \geq \bar{x}} x_{kt} d\mu_t = \gamma^* \mathbb{E}[\mu_t^e] \end{aligned}$$

where the inequality is determined by Proposition 1 and Equation (9). But then  $\mathbb{E}[\mu_{t+1}^e] < \gamma^* \mathbb{E}[\mu_t^e]$  and  $\mathbb{E}[\mu_{t+1}^e] = \gamma^* \bar{x}$  which is a contradiction since  $\bar{x} \geq \mathbb{E}[\mu_t^e]$ . This completes the proof.  $\square$

Since  $\bar{x} < \mathbb{E}[\mu_t^e]$ , then we have that  $\mathbb{E}[\mu_{t+1}^e] = \max_{x'} \{x' \in \arg \max_y u(\bar{x} - y/w^*) + \beta v(y, y)\} < \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_t^e] - y/w^*) + \beta v(y, y)\} = \mathbb{E}[\mu_t^e]$ , proving that also if  $\mu_{t+1}^e$  is not of perfect equality, then  $e_t$  has an average consumption that grows strictly more than  $e$ .

We have therefore showed that  $e_t$  has an average consumption that grows strictly more than  $e$  between time  $t$  and time  $t+1$  for all  $t$  such that  $\mu_t^e$  is not of perfect equality. Observe also that if we take two economies  $E'_1, E''_1 \in \mathcal{E}_1$  such that the initial endowment of  $E'_1$  is strictly higher than that of  $E'_2$ , then we have  $E'_1 \succ_{\max} E''_1$ . These two observations jointly imply  $E_1 \succ_{\max} E$  for all  $E \in \mathcal{E}_2 \cup \mathcal{E}_3$ .

We are left to analyze the case in which  $u$  is CRRA.

**Claim 3.** If  $u$  satisfies CRRA, then  $E_3 \geq E_0$  and  $E_2 \succ E_0$ .

*Proof.* Notice first of all that  $E_0$  has a unique equilibrium  $E_0$ , and that if  $u$  exhibits CRRA, then the growth rate of  $E_0$  is constant, and use  $\lambda$  to define it. We need to prove that  $e$  never grows less than  $\lambda$ . Call  $\lambda^*$  the growth rate of  $E_0$  the first period, and divide the population  $L$  into four groups:

- A: Subjects whose initial endowments is above or equal the average endowment;
- B: Subjects whose initial endowments is below the average endowment, but who meet their aspirations in the second period, i.e.  $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) = x_0(1 + \hat{\lambda})$ ;
- C: Subjects whose initial endowments is below the average endowment, and who do not meet their aspirations in the second period but have a second period consumption in a point where  $v$  is convex, i.e.  $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) \in [x_0(1 + \hat{\lambda} - \gamma, x_0(1 + \hat{\lambda})]$ ;
- D: Subjects whose initial endowments is below the average endowment, and who do not meet their aspirations in the second period but have a second period consumption in a point where  $v$  is concave, i.e.  $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) < x_0(1 + \hat{\lambda}) - \gamma$ ;

(Some of the groups above might be empty.) Define by  $\lambda_i^*$  the growth rate of the average consumption of each of the groups above, for  $i = A, B, C, D$ . Notice first of all that we must have  $\lambda_A^* \geq \lambda$ . Notice that every subjects in either group would increase her consumption of exactly  $\lambda$  if she had  $v = u$  instead of being reference dependent. Now, if  $\lambda^* \leq \lambda$ , then if subjects of group (a) increased their consumption of  $\lambda$ , they would remain strictly above their aspirations: but since above the aspirations

$u' = v'$ , increasing the consumption of  $\lambda$  must remain optimal for them. Therefore,  $\lambda_A^* \geq \lambda$ . Consider now subjects in B, and notice that they have an initial endowment below the average endowment, but a second period consumption exactly equal to the mean. This means that subjects of group B must have  $\lambda_B^* > \lambda^*$ . Consider now subjects in group C: were they not reference-dependent, they would increase their consumption of  $\lambda$ . And, their second-period consumption must lead is in an area in which the second period utility  $v$  is convex. But exactly since  $v$  is convex, they consume more in the second period then they would have if they were not reference-dependent –  $v$  coincides with  $u$  until a point after which it goes *above*  $u$  and becomes convex. Therefore, we must have  $\lambda_C^* > \lambda$ . Finally, consider the subjects in group D. Notice that, among them, we cannot have subjects such that, if they increased their consumption of  $\lambda$ , they would have a second period consumption in a point where  $v$  is convex, i.e.  $x_{i,0}(1 + \lambda) \in [x_0(1 + \hat{\lambda} - \gamma, x_0(1 + \hat{\lambda}))$ , but that instead increase it less, so that  $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) < x_0(1 + \hat{\lambda}) - \gamma$ . The reason, just like subjects in group C, if increasing the consumption of  $\lambda$  were optimal with no aspirations, it is even more so now, with aspirations, since  $v$  is strictly above  $u$  from  $x_0(1 + \hat{\lambda}) - \gamma$  on. Therefore, the only subjects in group D must be those for whom  $x_{i,0}(1 + \lambda) < x_0(1 + \hat{\lambda}) - \gamma$ . But since  $v$  coincides with  $u$  before  $x_0(1 + \hat{\lambda}) - \gamma$ , then these subjects must increase their consumption of at least  $\lambda$ .

We have just proved that: if  $\lambda^* \leq \lambda$  then  $\lambda_A^* \geq \lambda$ ;  $\lambda_B^* > \lambda^*$ ;  $\lambda_C^* \geq \lambda$ ;  $\lambda_D^* \geq \lambda$ . Clearly this implies that we cannot have  $\lambda^* < \lambda$ , hence  $\lambda^* \geq \lambda$ . Notice, moreover, that if group B were not empty, this would imply  $\lambda^* > \lambda$ .

The argument above must hold true for all periods, i.e. the growth rate of  $e$  must be above  $\lambda$  for all periods. This means  $E_3 \supseteq E_0$  and  $E_2 \supseteq E_0$ . We are left to show that  $E_2 \supset E_0$ . To see why, notice that  $E_2$  is characterized by the fact that, at some period  $t$ , group B above must be non-empty – the distribution must become of perfect equality, which implies that is a period in which some subjects ‘jump’ from being below to being at the average consumption. We have already argued that this implies that the growth rate must then be strictly above  $\lambda$ , proving the claim.  $\square$

Finally, notice that Claim 3 together with (2) imply (4) and (6) of Theorem 3.

*Q.E.D.*

## References

- AGHION, P., E. CAROLI, AND C. GARCÍA-PEÑALOSA (1999): “Inequality and Economic Growth: The Perspective of the New Growth Theories,” *Journal of Economic Literature*, 37, 1615–1660.
- APPADURAI, A. (2005): “The Capacity to Aspire,” in *Culture and Public Action*, ed. by V. Rao and M. Walton, The World Bank. Washington DC.
- BANERJEE, A. (1990): “Envy,” in *Economic Theory and Policy: Essays in Honour of Dipak Banerjee*, ed. by B. Dutta, S. Gangopadhyay, D. Mukherjee, and D. Ray, Bombay: Oxford University Press.
- BOWMAN, D., D. MINEHART, AND M. RABIN (1999): “Loss aversion in a consumption–savings model,” *Journal of Economic Behavior and Organization*, 38, 155–178.
- CAMERER, C. (1995): “Individual decision making,” in *Handbook of Experimental Economics*, ed. by A. R. J. Kagel, Princeton University Press, Princeton, vol. 2.



- CHATEAUNEUF, A. AND P. WAKKER (1999): “An Axiomatization of Cumulative Prospect Theory for Decision Under Risk,” *Journal of Risk and Uncertainty*, 18, 137–145.
- COOPER, B., C. GARCÍA-PENALOSA, AND P. FUNK (2001): “Status effects and negative utility growth,” *Economic Journal*, 642–665.
- CORNEO, G. AND O. JEANNE (2001): “Status, the distribution of wealth, and growth,” *The Scandinavian Journal of Economics*, 283–293.
- DALTON, P. AND S. GHOSAL (2009 (Forthcoming)): “Decisions with Endogenous Preference Parameters,” *Social Choice and Welfare*.
- DIECIDUE, E. AND J. VAN DE VEN (2008): “Aspiration level, probability of success and failure, and expected utility,” *International Economic Review*, 49, 683–700.
- FERSHTMAN, C., K. MURPHY, AND Y. WEISS (1996): “Social status, education, and growth,” *Journal of Political Economy*, 108–132.
- GENICOT, G. AND D. RAY (2009): “Aspirations, Inequality, Investment and Mobility,” *Georgetown University and New York University, mimeo*.
- HOPKINS, E. AND T. KORNIENKO (2006): “Inequality and Growth in the Presence of Competition for Status,” *Economics Letters*, 93, 291–296.
- KAHNEMAN, D. AND A. TVERSKY (1979): “Prospect theory: an analysis of choice under risk,” *Econometrica*, 47, 263–291.
- KONRAD, K. (1992): “Wealth seeking reconsidered,” *Journal of Economic Behavior and Organization*, 18, 215–27.
- KÖSZEGI, B. AND M. RABIN (2006): “A Model of Reference-Dependent Preferences,” *Quarterly Journal of Economics*, 121, 1133–1165.
- KOSZEGI, B. AND M. RABIN (2009): “Reference-dependent consumption plans,” *American Economic Review*, 99, 909–936.
- MASATLIOGLU, Y. AND E. A. OK (2005): “Rational Choice with status quo bias,” *Journal of Economic Theory*, 121, 1–29.
- (2008): “A Canonical Choice Model with Initial Endowment,” *Mimeo*, New York University.
- MOOKHERJEE, D., S. NAPEL, AND D. RAY (2010): “Aspirations, Segregation and Occupational Choice,” *Journal of the European Economic Association*, 8, 139–168.
- OK, E. A., P. ORTOLEVA, AND G. RIELLA (2009): “Revealed (P)Reference Theory,” *Mimeo*, New York University.

- ORTOLEVA, P. (2010): “Status Quo Bias, Multiple Priors and Uncertainty Aversion,” *Games and Economic Behavior*, 69, 411–424.
- RAUSCHER, M. (1997): “Conspicuous consumption, economic growth, and taxation,” *Journal of Economics*, 66, 35–42.
- RAY, D. (2006): “Aspirations, Poverty and Economic Change,” *In Understanding Poverty* (eds. A. Banerjee, R. Bénabou and D. Mookherjee), New York: Oxford University Press.
- SAMUELSON, W. AND R. ZECKHAUSER (1988): “Status quo bias in decision making,” *Journal of Risk and Uncertainty*, 1, 7–59.
- STARK, O. (2006): “Status Aspirations, Wealth Inequality, and Economic Growth,” *Review of Development Economics*, 10, 171–176.
- TVERSKY, A. AND D. KAHNEMAN (1974): “Judgment under uncertainty: Heuristics and biases,” *Science*, 185, 1124–1131.
- (1981): “The framing of decisions and the psychology of choice,” *Science*, 211, 453–458.
- (1991): “Loss Aversion in Riskless Choice: A Reference-Dependent Model,” *Quarterly Journal of Economics*, 106, 1039–1061.
- (1992): “Advances in prospect theory: cumulative representation of uncertainty,” *Journal of Risk and Uncertainty*, 5, 297–323.
- WAKKER, P. (2010): *Prospect Theory for Risk and Ambiguity*, Cambridge University Press.